



GCSE

REVISION

PACK

Number

Key Words:

Integer - A whole number

Improper Fraction - Fraction where the numerator is greater than the denominator, e.g., $\frac{6}{5}$

Mixed number - A number and a fraction written next to each other, e.g., $3\frac{1}{4}$

Inverse - Opposite

BIDMAS - Way of remembering the order to do operations in (Brackets, Indices, Division & Multiplication, Addition & Subtraction)

Prime Number - A number that is only divisible by itself and 1.

Reciprocal - 1 over a number, or the flipped version of a fraction, e.g., $\frac{2}{5}$ and $\frac{5}{2}$. A number multiplied by its reciprocal is always equal to 1.

Factor/Divisor - A number that divides exactly into a number, e.g., 3 is a factor of 9

Multiple - A number that is in the times table of another number, e.g., 15 is a multiple of 3

Highest Common Factor (HCF) - The highest number that can divide exactly into two numbers

Lowest Common Multiple (LCM) - The lowest number that is a multiple of two numbers

Surd - An exact square root of a number, e.g., $\sqrt{5}$

Index/Indices - Powers of a number, e.g., 2^3

Prime factorisation - Writing a number as a product of its prime factors

Standard Form - A number written in the form $A \times 10^n$ where A is between 1 and 10, and n is an integer.

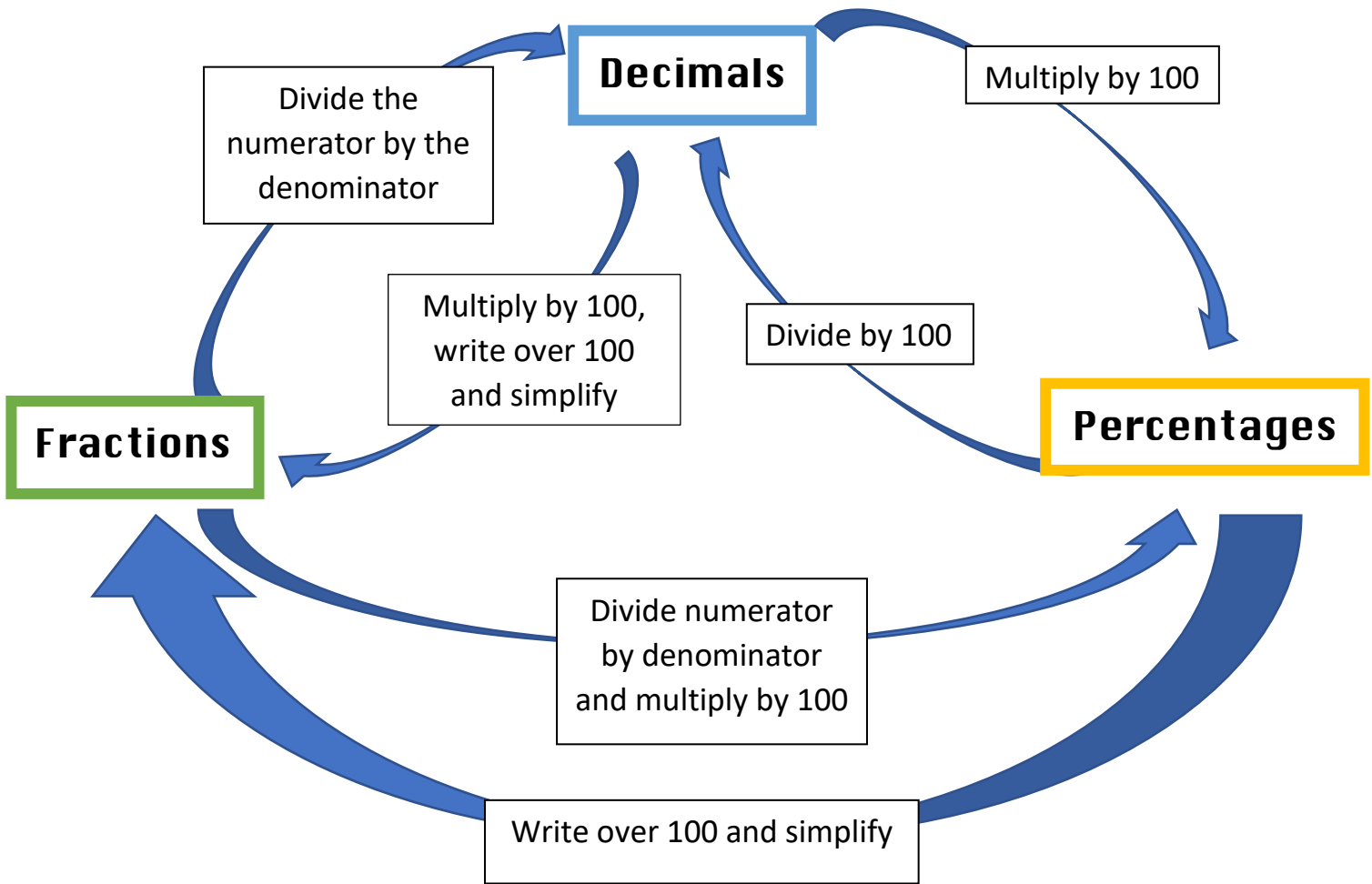
Product - Another word for multiplication

Sum - Another word for add

Times Tables

x	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

Converting between Fractions, Decimals, Percentages



Laws of Indices

$$(a^n)^m = a^{nm}$$

$$(ab)^n = a^n b^n$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$a^n a^m = a^{n+m}$$

$$a^{-n} = \frac{1}{a^n}$$

$$\frac{a^n}{a^m} = a^{n-m}$$

Order of operations

B (brackets)

I indices²

D ÷ division

M multiplication \times

A + addition

S subtraction $-$

equally important

equally important

Square Numbers

$1^2 = 1$

$5^2 = 25$

$9^2 = 81$

$13^2 = 169$

$2^2 = 4$

$6^2 = 36$

$10^2 = 100$

$14^2 = 196$

$3^2 = 9$

$7^2 = 49$

$11^2 = 121$

$15^2 = 225$

$4^2 = 16$

$8^2 = 64$

$12^2 = 144$

$16^2 = 256$

Cube Numbers

$$1^3 = 1$$

$$2^3 = 8$$

$$3^3 = 27$$

$$4^3 = 64$$

$$5^3 = 125$$

Unit Conversions

$$1kg = 1000g$$

$$1km = 1000m$$

$$1 \text{ meter} = 1000mm$$

$$1 \text{ meter} = 100cm$$

$$1cm = 10mm$$

$$1 \text{ Litre} = 1000ml$$

Geometry and Measures

Key words:

Perpendicular Lines - Lines that are at right angles to each other.

Parallel Lines - Lines that never meet.

Polygon - A shape with at least three straight lines.

Regular Polygon - A polygon where all the sides are the same length and all the angles are the same size.

Area - The size of a shape.

Perimeter - The length around the outside of a shape.

Equilateral (Triangle) - A triangle where all the sides are the same length.

Isosceles (Triangle) - A triangle where two of the sides are the same length.

Scalene (Triangle) - A triangle where all the sides are different lengths.

Right Angled (Triangle) - A triangle which contains a right angle.

Vertex - A point; where two lines or edges meet.

Circumference - The perimeter of circle; The length around the outside of a circle.

Radius - Distance between the centre of the circle and the outside of the circle.

Diameter - Distance across the middle of a circle through the centre.

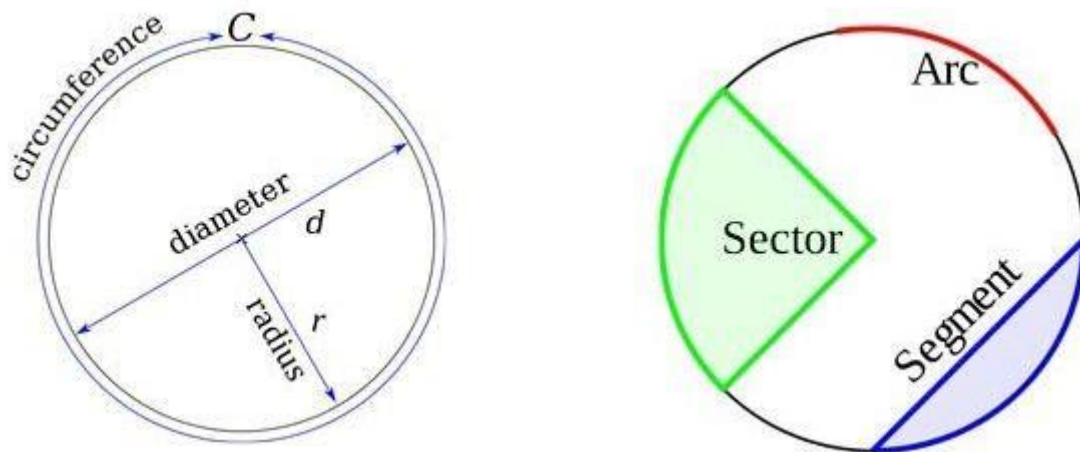
Chord - A line which joins two lines on the circumference on a circle. The longest possible chord is the diameter.

Bearing - The angle as measured from north.

Vector - A quantity with direction and size, often written in a column like $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$.

Hypotenuse - The longest side of a right-angled triangle.

Circle Facts



Area of a circle = πr^2

Circumference of a circle = πd or $2\pi r$

Arc Length = $\frac{\text{angle}}{360} \times \pi d$

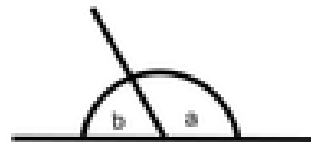
Sector Area = $\frac{\text{angle}}{360} \times \pi r^2$

Angle Type	Description
Acute	Less than 90°
Right Angle	90°
Obtuse	Between 90° and 180°
Reflex	More than 180°

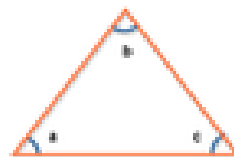
Angle Types

Angle Facts

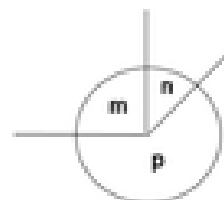
The angles on a straight line add up to 180° .
 $a + b = 180^\circ$



The angles in a triangle add up to 180° .
 $a + b + c = 180^\circ$



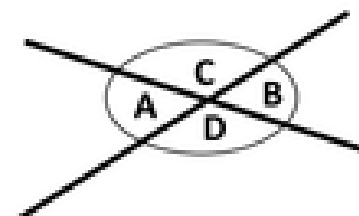
The angles at a point add up to 360° .
 $m + n + p = 360^\circ$



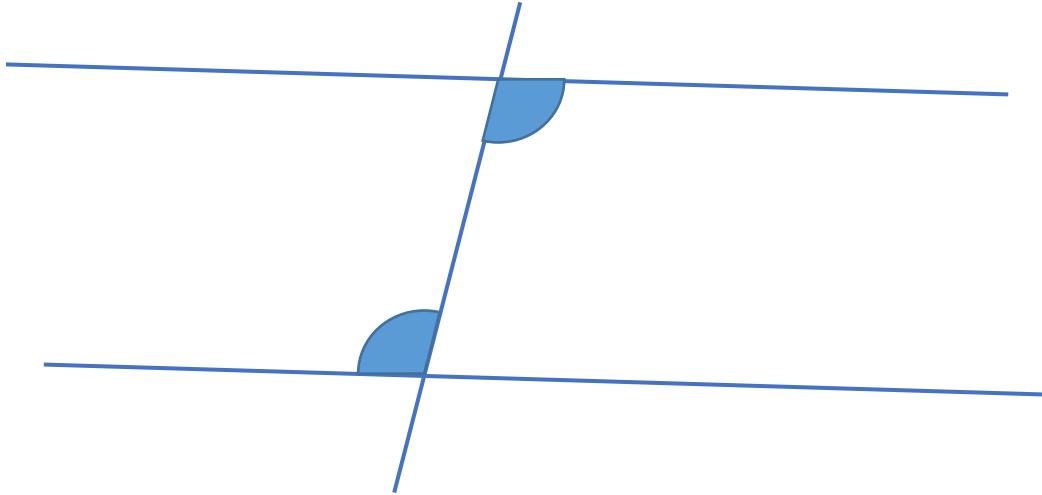
The angles in a quadrilateral add up to 360° .
 $w + x + y + z = 360^\circ$



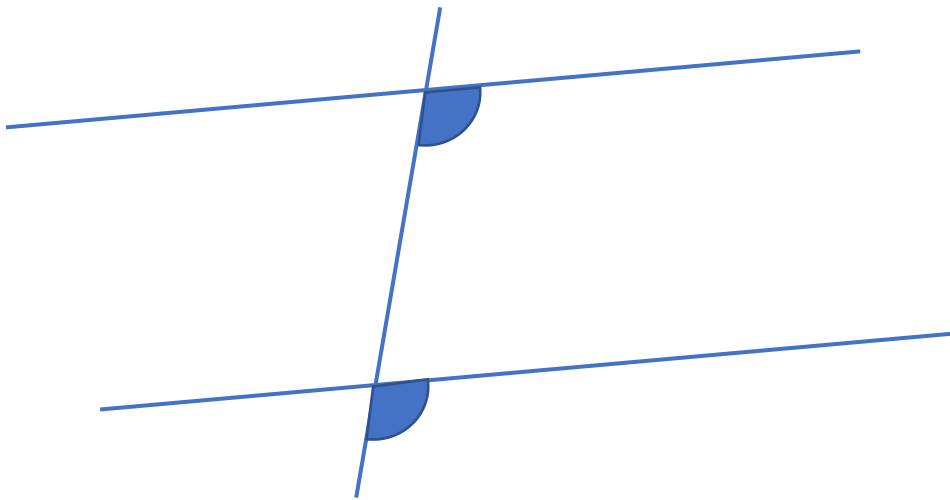
Vertically opposite angles are equal.
 $A = B$
 $D = C$



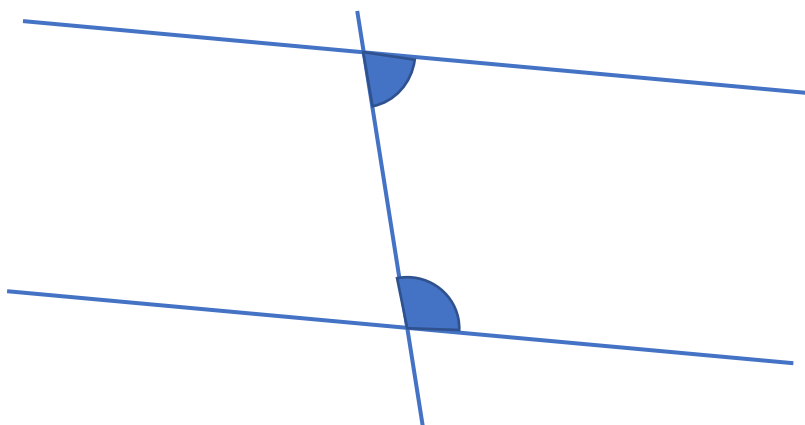
Alternate Angles (Z angles) are equal



Corresponding Angles (F angles) are equal

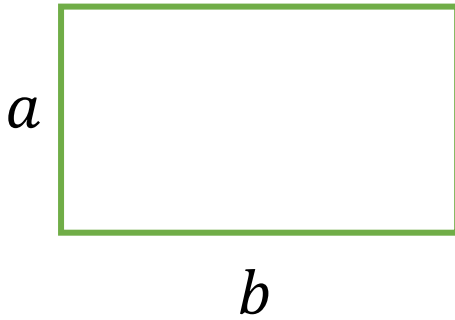


Co-Interior Angles (C angles) add up to 180°



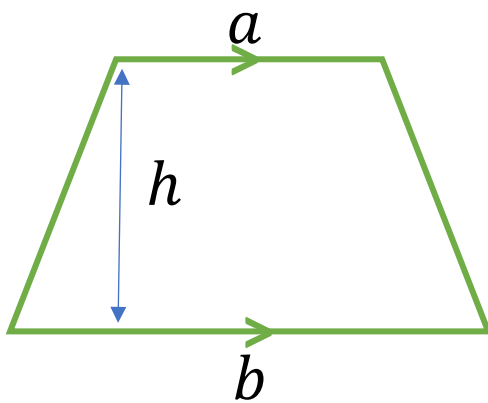
2D Shapes

Rectangle



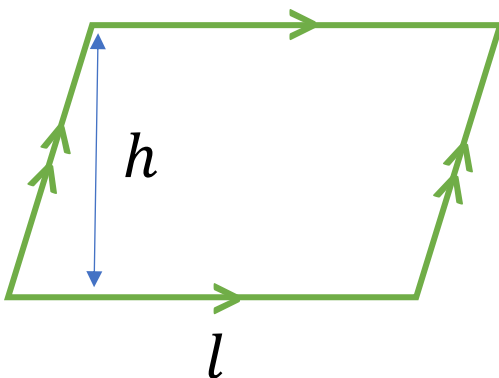
$$\begin{aligned} \text{Area} &= \text{base} \times \text{height} \\ &= a \times b \end{aligned}$$

Trapezium



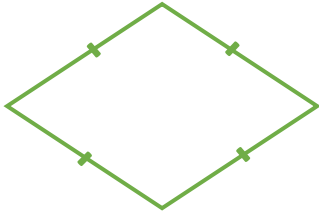
$$\text{Area} = \frac{a + b}{2} \times h$$

Parallelogram

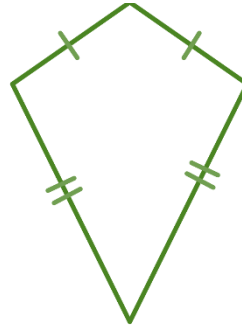


$$\text{Area} = l \times h$$

Rhombus

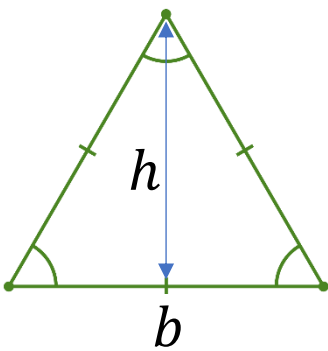


Kite

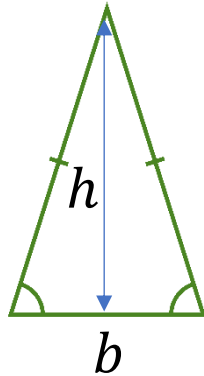


Triangles

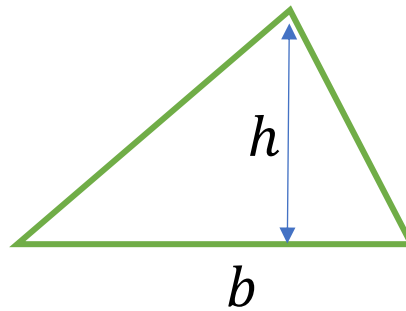
Equilateral



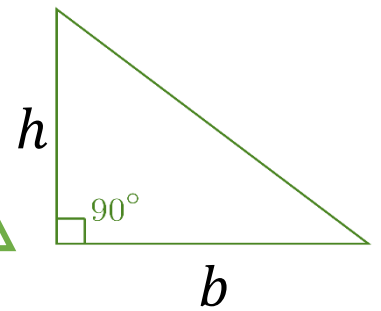
Isosceles



Scalene



Right angle



The height here is always the perpendicular height!

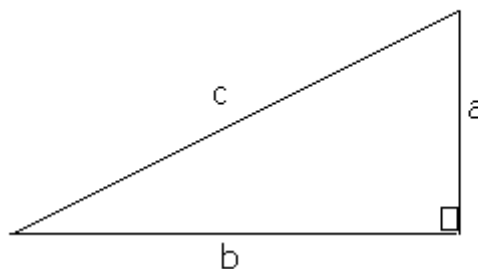
$$\begin{aligned} \text{Area of a Triangle} &= \frac{\text{Base} \times \text{Height}}{2} \\ &= \frac{b \times h}{2} \end{aligned}$$

Pythagoras'

Theorem:

$$a^2 + b^2 = c^2$$

Where c is the hypotenuse



Trigonometry:

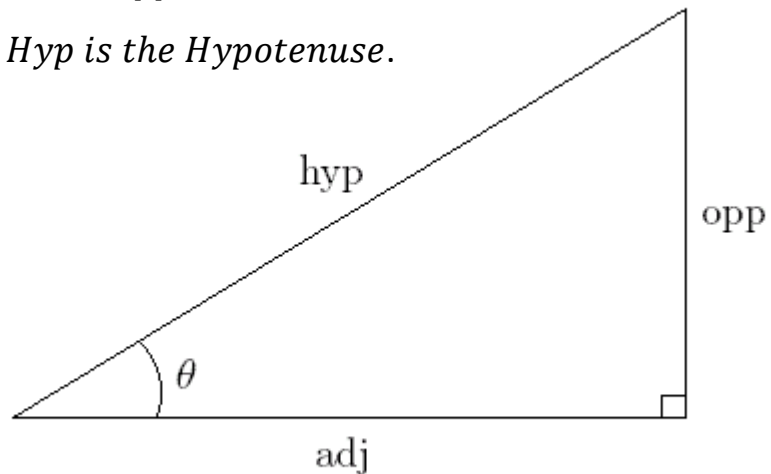
In a right-angled triangle with an angle θ , the trigonometric ratios are

$$\sin\theta = \frac{Opp}{Hyp} \quad \text{where Adj is the Adjacent side,}$$

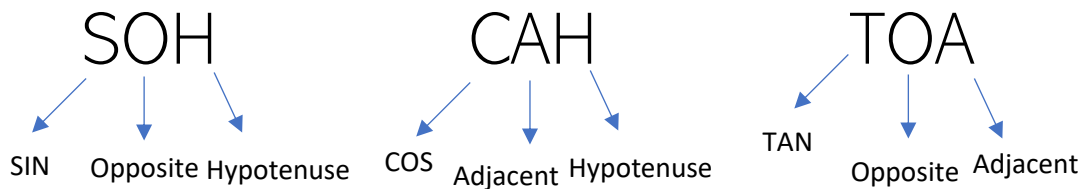
Opp is the Opposite side,

$$\cos\theta = \frac{Adj}{Hyp} \quad \text{and Hyp is the Hypotenuse.}$$

$$\tan\theta = \frac{Adj}{Opp}$$

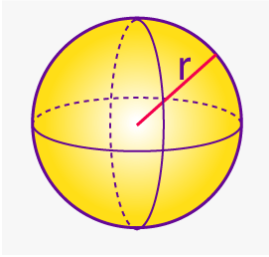


To remember the ratios, remember



3D Shapes

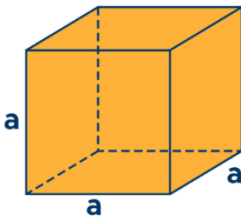
Sphere



$$\text{Volume} = \frac{4}{3}\pi r^3$$

$$\text{Surface Area} = 4\pi r^2$$

Cube



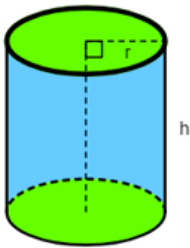
$$\text{Volume} = a \times a \times a = a^3$$

$$\text{Surface Area} = 6a^2$$

Prism

$$\text{Volume} = \text{Area of Cross Section} \times \text{Length}$$

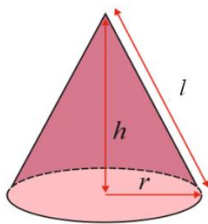
Cylinder



$$\text{Volume} = \pi r^2 h$$

$$\text{Surface Area} = \pi d h + 2\pi r^2$$

Cone



$$\text{Volume} = \frac{1}{3}\pi r^2 h$$

$$\text{Surface Area} = \pi r l + \pi r^2$$

ALGEBRA

Key Words:

Solve - Work out the value of the letter.

Variable/Unknown - A symbol for a value we don't know yet.

Coefficient - Number in front of a variable

Factorise - Put brackets in, e.g $3x + 9 = 3(x + 3)$

Term - A single number or variable, or numbers and variables multiplied together.

$4, 2x, -\frac{1}{x}, 3ab$ are all terms.

Expression - A set of terms combined using the operations $+, -, \times$ and \div , for example $4x - 1$ or $x^2 + 2xy + 2$.

Equation - A statement showing two expressions are equal, often to be solved for an unknown, for example $2x + 1 = 3$ or $4x - 1 = x + 7$

Identity - A statement that is always true no matter which values are chosen. For example, $4a \times a^2 = 4a^3$

Formula - A mathematical relationship expressed in symbols, for example the formula for the Area of a circle, $A = \pi r^2$.

Gradient - The slope of a straight line, worked out by $\frac{\text{change in } y}{\text{change in } x}$.

Y-Intercept - Where a straight line crosses the y- axis.

Simultaneous Equations - Two equations in two variables to be solved at the same time, for example

$$(1) 4x + y = 12$$

$$(2) 2x - 2y = 6$$

Inequality - Like an equation, except instead of an equals sign, there is a greater than (or equal to) or less than (or equal to). For example, $4x + 2 < 8$ or $2x - 1 \geq 9$

Sequence - A set of numbers that follow a pattern or a rule.

Notation

We write

$$a \times b \text{ as } ab$$

$$3 \times y \text{ and } y + y + y \text{ as } 3y$$

$$a \times a \text{ as } a^2$$

$$a \div b \text{ as } \frac{a}{b}$$

Collecting Like terms

If a question asks you to 'collect like terms', it means you group together all the terms that can be grouped together. For example

$$4x + 2y - x - 4y$$

becomes

$$3x - 2y$$

The x terms are like terms, so are grouped together,
 $4x - x = 3x$

The y terms are like terms, so are grouped together,
 $2y - 4y = -2y$

x and x^2 are not like terms and cannot be grouped together! This is a common mistake!

$$4(x + 2) = 4x + 8$$

Expanding Brackets

We can then collect like terms!

$$(x + 2)(x + 3) = x^2 + 2x + 3x + 6$$

Straight Line Graphs

The general equation of a straight line

$$y = mx + c$$

c is the y-intercept

m is the gradient

- m is the out by the slope of the straight line.
 - c is the y-intercept and is where the line crosses the y-axis.
- gradient and can be figured doing $\frac{y \text{ increase}}{x \text{ increase}}$. The gradient is

Solving Equations

Golden Rules for Solving Equations:

- Whatever you do to one side, you must do to the other
- The aim is to get the unknown on it's own on one side of the equation.
- Use inverse operations to get the unknown on it's own, remember addition and subtraction are inverses, and multiplication are inverses.

Probability

Key Words

Probability - The chances of something happening.

Relative Frequency - The number of times an event happens divided by the total number of trials in an experiment.

Biased - Unfair or skewed towards a certain outcome.

Systematic Listing - Listing the outcomes of an event in an organised way.

Mutually Exclusive - Events that cannot happen at the same time are mutually exclusive.

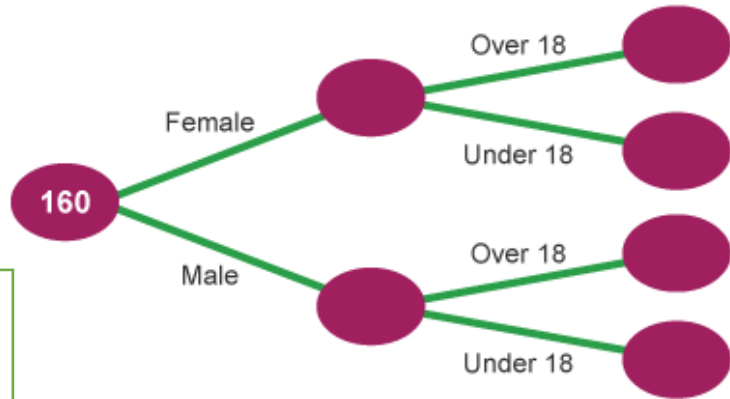
Independent Events - Two events are independent if the probability of one event does not affect the probability of another.

The probability of an outcome = $\frac{\text{the number of ways the outcome can happen}}{\text{total number of possible outcomes}}$

Use systematic listing to list the total possible outcomes

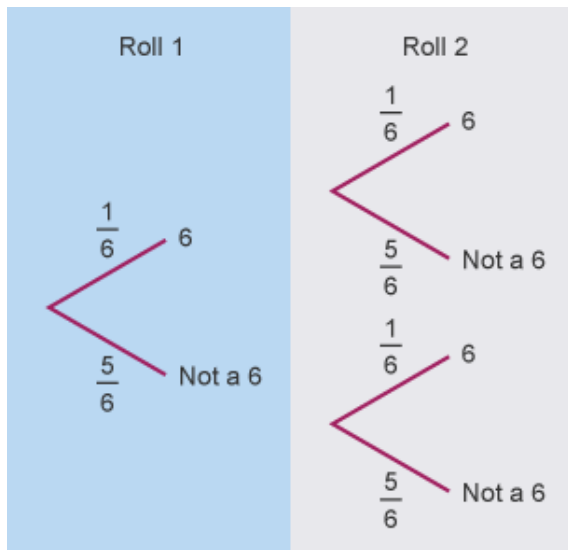
Frequency Trees

A frequency tree can be used to record and organize information given as frequency. This can then be used to calculate probabilities.



Tree Diagrams

Tree Diagrams are a visual way of showing all possible outcomes of two or more events. Each branch is a possible outcome and is labelled with a probability



6, 6

6, not a 6

Not a 6, 6

Not a 6, not a 6

In this example there are four possible outcomes. To work out the possibilities of each outcome just multiply the probabilities together.

$$6, 6 = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$$

$$6, \text{ not a } 6 = \frac{1}{6} \times \frac{5}{6} = \frac{5}{36}$$

$$\text{Not a } 6, 6 = \frac{5}{6} \times \frac{1}{6} = \frac{5}{36}$$

$$\text{Not a } 6, \text{ not a } 6 = \frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$$

Independent Events

If two events are independent, this means the outcome of one does not affect the outcome of the other. If events are independent, then we can use the following rule:

If we have two events A AND B , then

$$P(A \text{ and } B) = P(A) \times P(B)$$

$$P(A \text{ or } B) = P(A) + P(B)$$

Statistics

Key words:

Population - The whole set of items of interest (This can be a population of people but can equally be something such as all the lightbulbs in a factory)

Sample - A group within the population intended to represent the population

Census - Data collected from the entire population

Simple Random Sample - A sample where everything in the population has an equal chance of being selected for the sample.

Mean - Add up all the numbers and divide by how many there are.

Median - Order the numbers and select the middle number. If there are two numbers in the middle, find the mean of these two numbers.

Mode - The most common number

Range - The biggest number take away the smallest number.

Outlier - A data point that differs massively from the rest of the data.

Correlation - The relationship between two sets of data.

Positive Correlation - As one set of data increases, the other set tends to increase.

Negative Correlation - As one set of data increases, the other set tends to decrease.

Interpolation - Using a trend of two sets of data to estimate data values, within the range of the data.

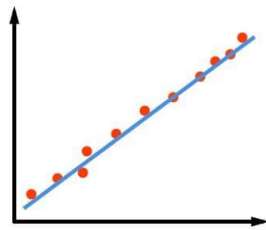
Extrapolation - Using a trend of two sets of data to estimate data values, outside the range of the data. Extrapolation is not always possible.

Primary Data - Data collected by the researcher.

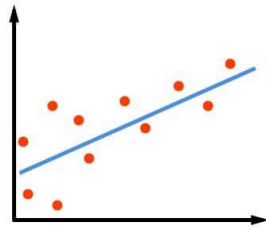
Secondary Data - Data not collected by the researcher, perhaps taken from national statistics.

CORRELATION

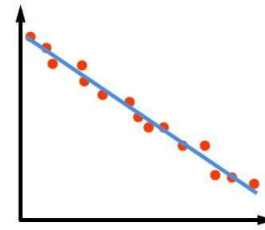
(INDICATES THE RELATIONSHIP BETWEEN TWO SETS OF DATA)



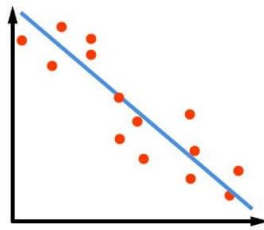
STRONG POSITIVE CORRELATION



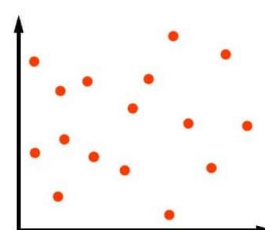
WEAK POSITIVE CORRELATION



STRONG NEGATIVE CORRELATION



WEAK NEGATIVE CORRELATION



NO CORRELATION

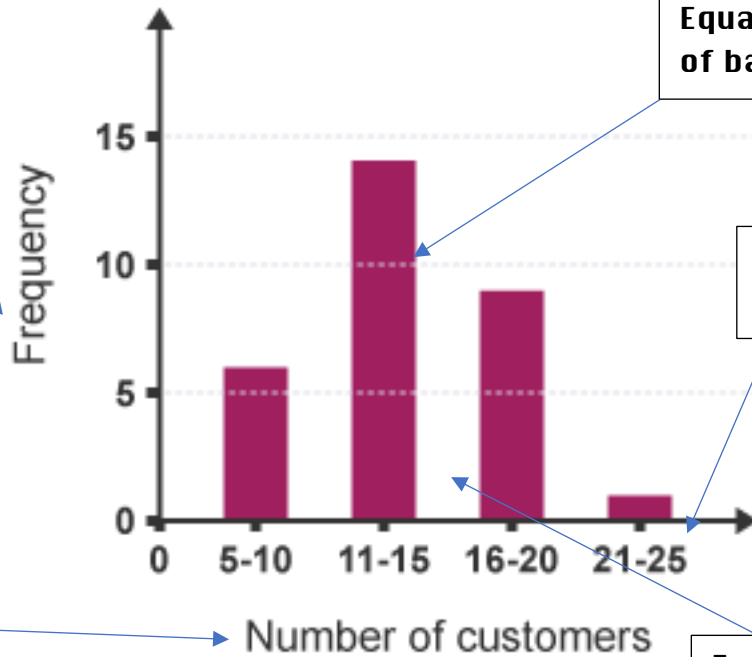
Ways of representing data

Frequency Table

Club	Tally	Frequency
Hockey		8
Badminton		7
Football		5

Used to show the frequency of certain things in a survey

Bar Chart



Label both axis

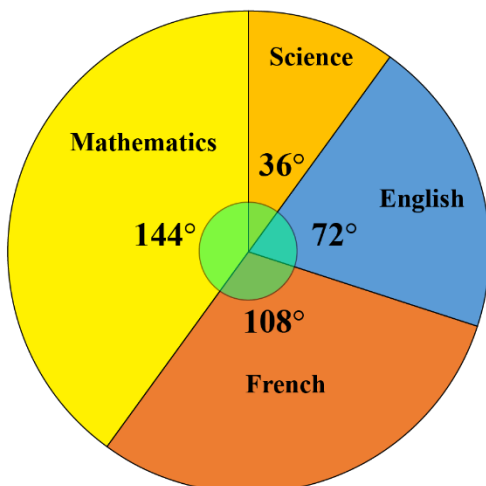
Equal widths of bars

Label the bars

Equal gaps between bars

When drawing a bar chart remember these important parts.

Favorite Subject



Pie Charts

In a Pie Chart, the proportion that each angle takes up is the proportion of the whole sample size that that category takes up.

For example, if in the example here 100 students were asked.....

the proportion of students whose favourite subject is maths is $\frac{144}{360} = \frac{2}{5}$ and $\frac{2}{5}$ of 100 is 40.

It is the biggest angle and so the most popular subject - taste!

Ratio and Proportion

Key Words:

Ratio - A way of showing how something is shared.

Numerator - The top number on a fraction

Denominator - The bottom number of a fraction

Common Denominator - A common denominator exists when the denominator (the number at the bottom of a fraction) of two or more fractions is the same. Common denominators help to compare or add/subtract two or more fractions.

Equivalent Fractions - Two fractions are equivalent when they mean the same thing but are written differently. $\frac{3}{5}$ is equivalent

to $\frac{6}{10}$ as the numerator and denominator have both been multiplied by 2.

Percentage Multipliers - A way of finding percentages by multiplying by a number.

Interest - money that is paid regularly at a particular percentage, usually when money has been lent or borrowed.

Compound Interest - Each time interest is paid onto an amount saved or owed, the added interest also receives interest from then on.

Scale Factor - A number which scales or multiplies a quantity, often used when talking about how many times bigger one shape is than another.

Finding percentages of amounts

There are many ways to find the percentage of an amount.

Find smaller percentages, e.g. 10%, 1% and then multiply up.

Write it as a fraction and then multiply the number by it.

Use a percentage multiplier

Remember, 'of' means multiply!

Using the example, Find 30% of 80

First find 10% of 80

To find 10% we divide by 10, so 10% of 80 = 8

To get 30%, we just multiply 10% by 3, so

$$30\% = 8 \times 3 = 24$$

$$30\% = \frac{30}{100} = \frac{3}{10}$$

So 30% of 80 is

$$\frac{3}{10} \times 80 = \frac{3}{10} \times \frac{80}{1}$$

$$= \frac{240}{10} = 24$$

To find the percentage multiplier we do $30 \div 100 = 0.3$

Then simply use a calculator to find $0.3 \times 80 = 24$

Percentage Multipliers

We can use percentage multipliers to find percentage increases or decreases. This is more useful when you have a calculator.

- 1) We work out what the overall percentage would be after the figure has had its percentage increase or decrease added or subtracted.
- 2) Then convert this amount to a decimal.
- 3) Finally multiplying by the number in question.

For example:

What is the multiplier for a 15% increase?

A 15% increase would mean that the overall percentage would be $100\% + 15\% = 115\%$

115% as a decimal = $115 \div 100 = 1.15$

So to find a 15% increase of a number, we would multiply by 1.15

For a 15% decrease, the overall percentage would be $100\% - 15\% = 85\%$, so the percentage multiplier would be 0.85

Finding a number as a percentage of another

To find a number as a percentage of another, divide the first number by the second and then multiply by 100.

Finding percentage change

Percentage change is calculated by dividing the difference between the two amounts by the original amount, for example:

If a shop was selling an item of clothing for £40 and is now selling it for £50, the price increase is $50 - 40 = 10$

£10 as a percentage of £40 is $\frac{10}{40} \times 100 = 25\%$

so this is the percentage increase.

Compound interest is where interest is made on money each year, and then that interest gains interest the following year.

For example, if you have £2000 in an account with 2% interest, the following year the account will have £2000 + 2% of £2000. To work this out we use the percentage multiplier 2000×1.02 .

For subsequent years we will be doing the same thing to this new amount, or to think of it another way, we'll be doing $2000 \times 1.02 \times 1.02$, or 2000×1.02^2

Following the pattern if we continue to leave this money in the account every year it will grow, and the amount in the account after n years will be

$$2000 \times 1.02^n$$

Compound Interest

In general:

If we invest £ P into an account with $r\%$ interest per year for n years, then the total money in the account at the end will be:

$$P\left(1 + \frac{r}{100}\right)^n$$

Finding ratios

If we want to split a number into a certain ratio, we have to think about how many parts we need to split that number up into.

If we split a number into the ratio 2:3:4 for example,



we can see that we have to split the number up into $2+3+4$ parts which is 9 parts. Then we can find out how much each part is worth.